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ABSTRACT

This document is the response manual for the learning module, "Teaching the Concept of Area." It contains the answers for the post-assessment tests given in the student manual. Instructor's manual and student manual available separately. (MB)

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RESPONSE MANUAL  
TEACHING THE CONCEPT OF AREA

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## Post-Assessment Element I

### Discussion of Your Responses:

The following are the items in the table:

Name of Item	Type of measurement
1. string	one - length two - cover
2. reel of wire	one - length two - cover
3. chicken wire	one - length two - cover
4. ruler	one - length two - cover
5. coins	one - length two - cover three - fill
6. sand	two - cover three - fill
7. kidney beans	one - length two - cover three - fill
8. water	two - cover three - fill
9. tiles	two - cover three - fill
10. cubes	three - fill
11. cups	three - fill
12. marbles	three - fill
13. index cards	two - cover three - fill
14. stamps	two - cover
15. sheets of paper	two - cover three - fill
16. board erasers	two - cover three - fill

(continued)

Name of item	Type of Measurement
17. bricks	two - cover three - fill
18. books	two - cover three - fill
19. foot prints	two - cover
20. leaves	two - cover three - fill
21. a roll of brown gummed tape	one - length two - cover
22. a roll of cotton 2" wide gauze	one - length two - cover

Discussion of your response to the selection of items for measurement  
of two dimensional space:

You should have included the fact in your answers that all your choices  
of items for measurement of two dimensional must be items which can be  
covering items.

For example: string  
Pieces of string placed side by side  
to cover a region.



ruler  
Rulers placed side by side to cover  
a region.

tiles  
Placed side by side to cover a region.

## Post-Assessment Element II

### Discussion of Your Responses:

The following are the items in the table:

Table II

Figure	Yes or No
a	yes
b	no
c	no
d	yes
e	yes
f	yes
g	yes
h	yes
i	yes
j	yes
k	yes
l	yes
m	no
n	yes
o	yes
p	no

a	yes
b	no
c	no
d	yes
e	yes
f	yes
g	yes
h	yes
i	yes
j	yes
k	yes
l	yes
m	no
n	yes
o	yes
p	no

### Post-Assessment Element III

#### Discussion of Your Responses to **Table III**

Answers will vary depending upon which plane region objects you have selected in Task 1.1. You should have at least fifteen plane region objects listed.

In the Standard Unit Column, you should have included - the square centimeter unit, the square inch unit, the square foot unit. Answers may vary. You may check the suggested readings given on page 60 for other standard units.

In the Non-Standard Unit Column, you may have many of the non-standard covering units from Post-Assessment Activity 1. For example, check your list from that Post-Assessment Activity 1.

#### Discussion of Your Response To B

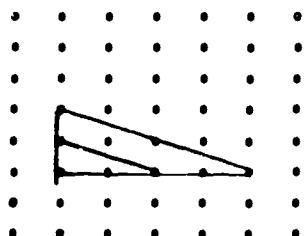
Your responses will vary but they should have the following ideas in them:

The use of a standard grid provides you with ability to obtain reasonable estimates of the areas of regular and irregular figures. Standard measurements will give one the opportunity to make comparisons between one or more regions.

**Post-Assessment Element IV**

**Task 4.2**

(1) Does your dot paper diagram look like this?

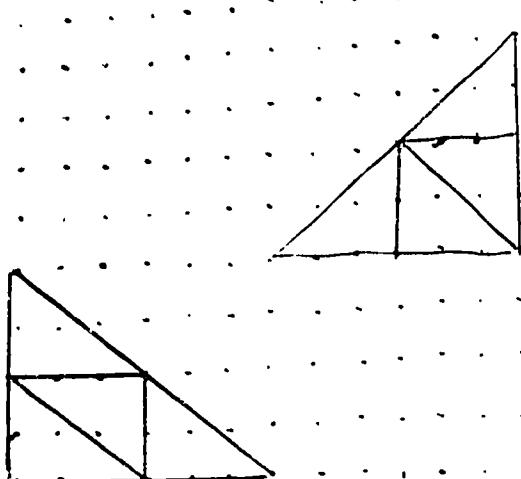


If yes, then continue and check your table 4.2c results.  
If not, see where you went wrong. Correct it and redo  
table 4.2c. If you are not sure why the board should  
look like this and you had trouble with (a) and/or (b)  
go to the instructor for further help.

Table 4.2c

	Base	Height	Area
Set IX	2	1	1
Set X	4	2	4

(2) Does your dot paper look like this?



**Post-Assessment Element IV**

**Task 4.2 (cont'd.)**

If yes, then continue to check your table 4.2c.

If no, go back and rework 4.2 (a) and (b) or read one of the suggested texts or see your instructor.

**Table 4.2c (cont'd.)**

	Base	Height	Area
Set XI	3	2	3
Set XTI	6	4	12

A possible response:

When the base and height units are each double then the number of square units in the area is multiplied by 4.

or

$$\text{Old Area} = \frac{1}{2}bh$$

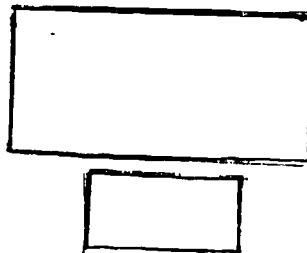
$$\begin{aligned}\text{New Area} &= \frac{1}{2} (2b) (2h) \\ &= 2 bh\end{aligned}$$

$$\text{Note: } 2bh = 4(\frac{1}{2}bh)$$

Post-Assessment Element IV

Task 4.3

Does your picture look like this?



If yes, continue. If no, go back and redo task 4.4 or read one of the suggested texts or see your instructor.

Table 4.3c

Whole	8	4	32
First Paste	4	2	8
Second Paste	2	1	2

When the length of a rectangle is halved and the width is halved the area of the new figure is one fourth the original area.

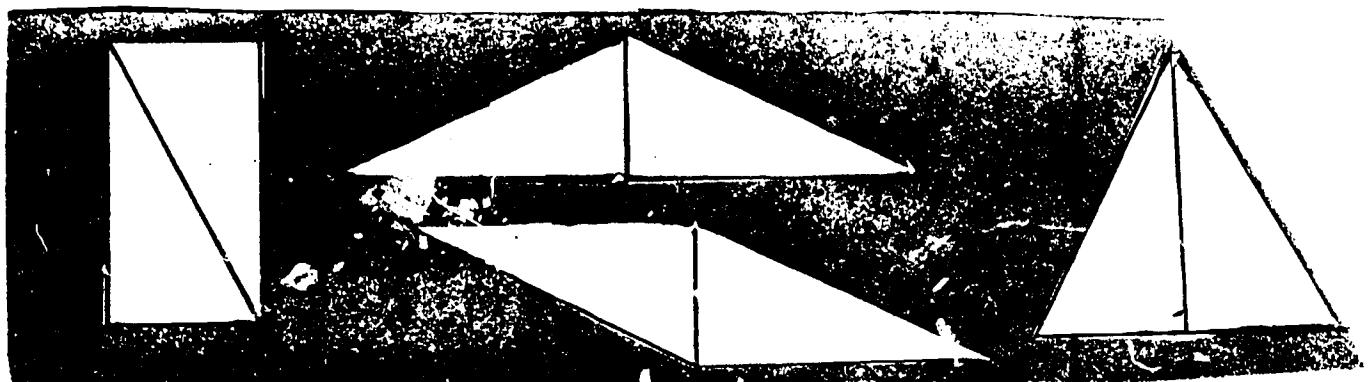
$$\text{Original Whole } A = l w$$

$$\begin{aligned}\text{New } A &= \frac{1}{2} l \times \frac{1}{2} w \\ &= \frac{1}{4} l w\end{aligned}$$

Post-Assessment Element V

Task I

(1) Does your paste picture look like the ones below?



Yes - Proceed to next part

No - Do Task 5c (2) and then come back to this part

Possible response:

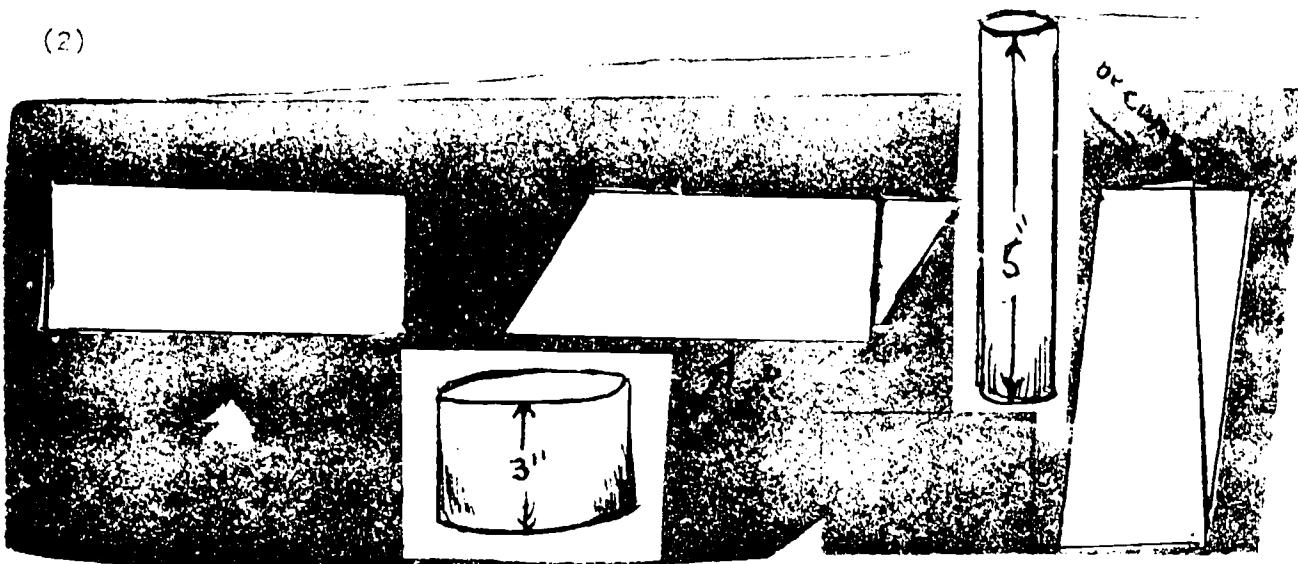
The 12 square centimeters fill the surface of each figure. They all are 12 centimeter squares in area

or

The figures look very different but all are equivalent in area

Post-Assessment Element V

(2)



Does your work look like this?

Yes - Go on

No - Go back, read directions again or get help

Possible responses:

(3) The figures all came from equivalent rectangular regions and are equivalent in area. The scotch taped part shows the triangular piece that can be cut off and moved to re-form with the original rectangular region.

This activity has emphasized that objects look different but have the same surface area.

(4) Dividing rectangular shapes into two parts in different ways can form triangles and parallelograms of equivalent areas. The check in area equivalence can be accomplished by placing chosen units in the region or by re-forming them to show the equivalence.

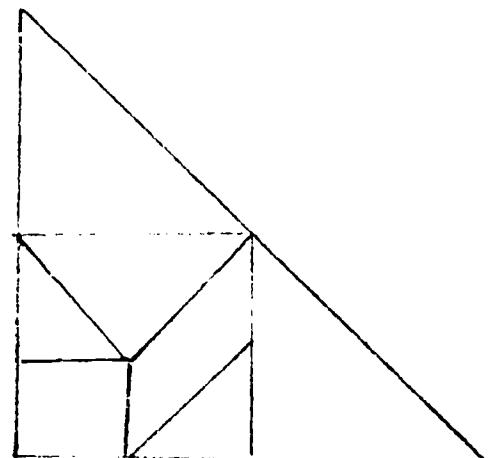
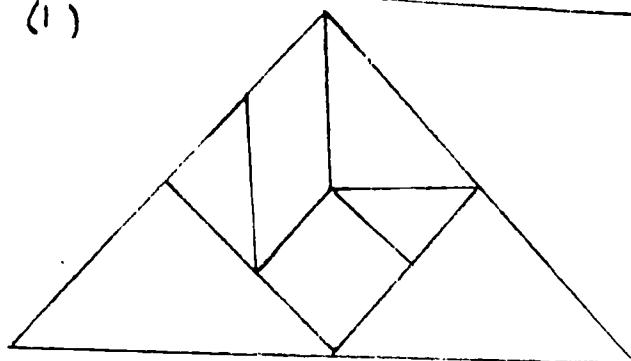
There is a need to experiment to form equivalent areas.

**Post-Assessment Element V**

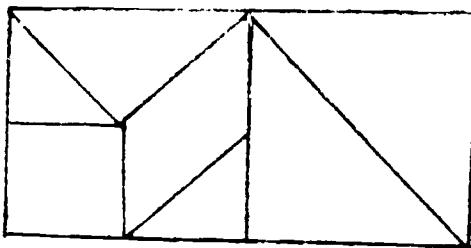
**Task II**

b. Some possible solutions are:

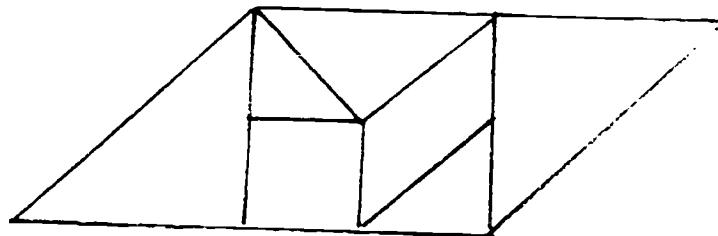
(1)



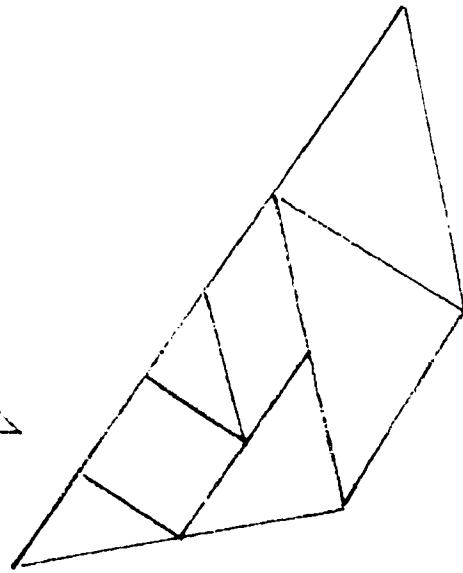
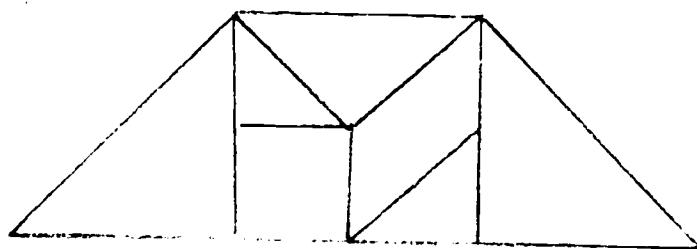
(2)



(3)

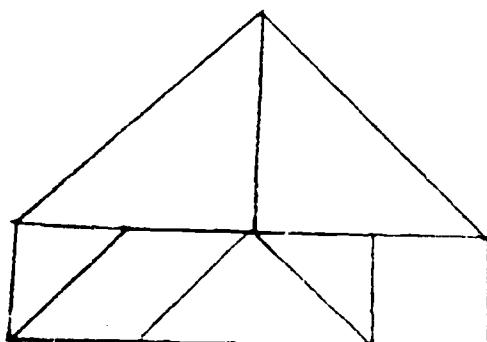


(4)

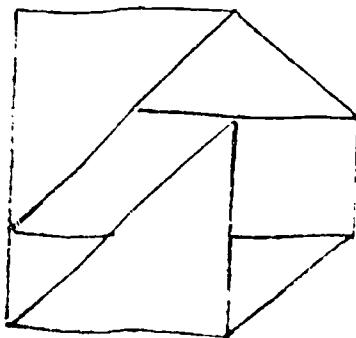
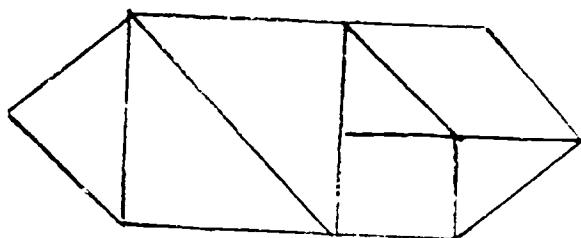


Task II b (cont'd.)

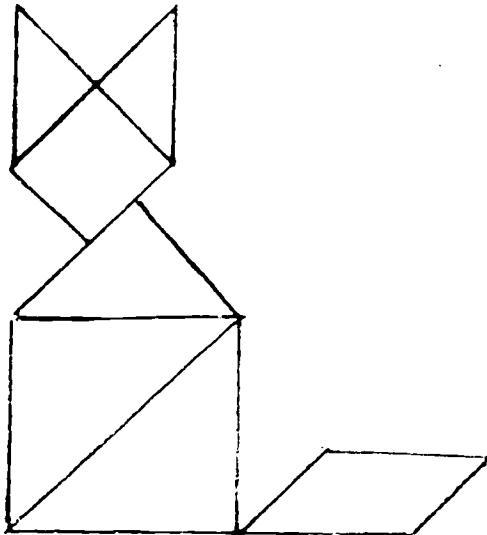
(5)



(6)



(7)



c. The areas of all the figures formed in (b) are equal

**Post-Assessment Element VI**

### Task I

a. Pick's Theorem

1. For a figure having b boundary points and no interior points, its area in square units is one less than one-half the number of boundary points.

$$\text{That is, } \text{area} = \frac{1}{2} b - 1$$

2. For a figure having  $b$  boundary points and  $i$  interior points, its area in square units is one less than the sum of the number of interior points and one-half the number of boundary points.

That is,  $\text{area} = i + \frac{1}{2}b-1$

$$\text{b. (1) Area of square } ABCD = 9 + \frac{1}{7}(4) = 10 \text{ square units}$$

$$(2) \text{ Area of triangle } XYZ = 9 + \frac{1}{2} (3) - 1 \\ = 9 \frac{1}{2} \text{ square units}$$

c. Area =  $9\frac{1}{2}$  square units      Area = 17 square units

d. Alternate methods include:

- (1) Dividing interior region up into triangles, rectangles or squares and finding area of each part;
- (2) Building a rectangle around the figure, finding the area of this rectangle and finding the area between the given figure and the rectangle.

(For further explanation see Discussion of Responses for Task 6.1 i and 6.4 g)

Post-Assessment Element VI

Task II

a. (1) larger  
smaller  
40 square units  
16 square units  
Between 16 square units and 40 square units

(2) equivalent  
16 square units  
12 square units

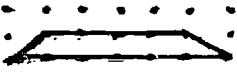
(3) equivalent  
16 square units  
4 square units  
8 square units

(4) equivalent  
20 square units  
8 square units

(5) 28 square units  
Yes

b. 15 square units for area of each trapezoid

c. Some possibilities include:



d. Some possible entries:

Table 6.8

Length of One Base	Length of Other Base	Average Length of two bases	Height	Area
8	4	6	4	16
6	2	4	4	16
5	3	4	3	12
10	1	5.5	2	11
5	1	3	6	18

Hypothesis:

Area of trapezoid is equal to product of average length of the bases and the height of the trapezoid.

Task II (cont'd.)

e. Area =  $m \times h$

f. Yes. Area of parallelogram =  $b \times h$

In a parallelogram, the average of the two bases would be  $\frac{b}{2}$

$$\text{i.e. } m = \frac{b+b}{2} = b$$

Hence area of parallelogram =  $m \times h$

Yes. Area of rectangle =  $m \times h$  for the same reason as parallelogram.

Yes. Area of triangle =  $\frac{1}{2} b \times h$

In a triangle, the upper base would have length of zero; thus average of two bases would be  $\frac{1}{2}b$

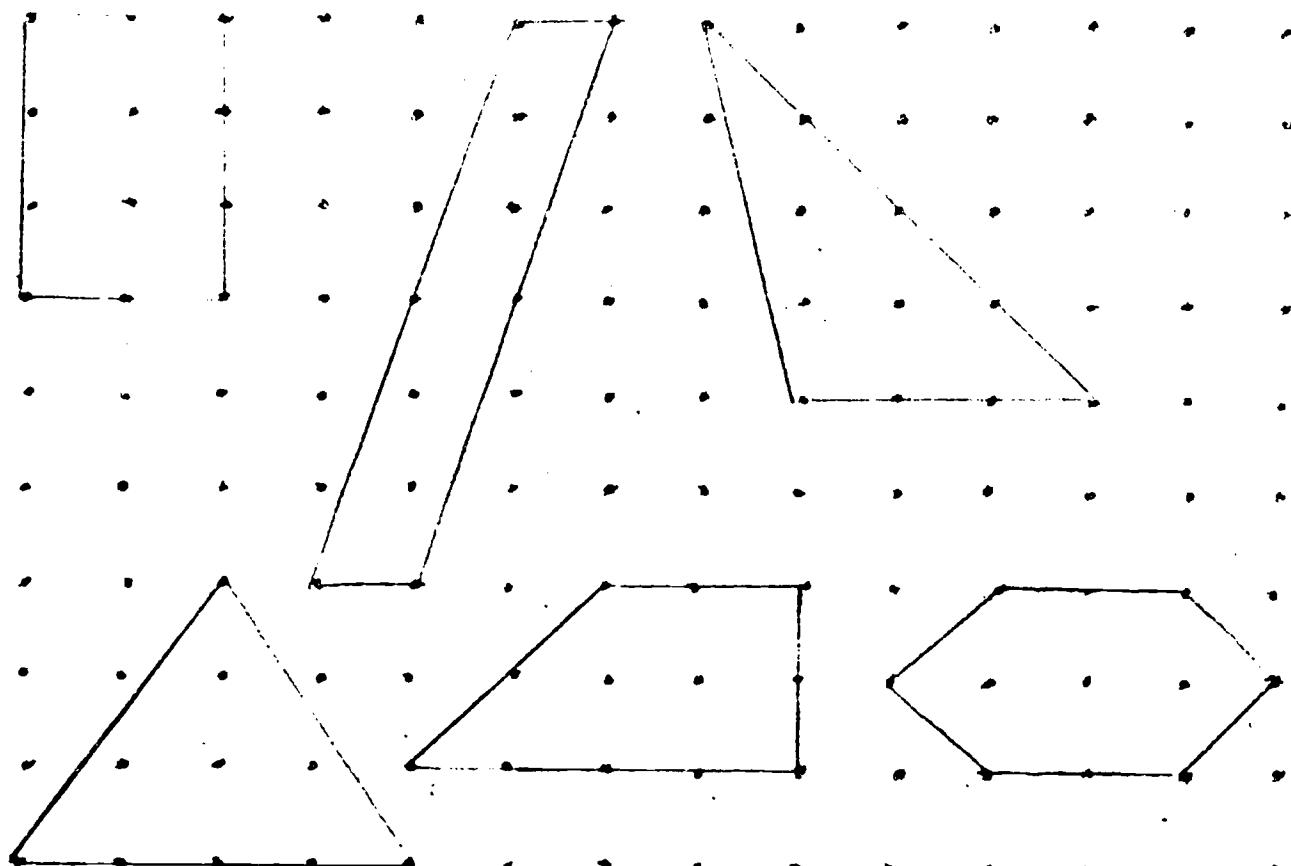
$$\text{i.e. } m = \frac{0+b}{2} = \frac{1}{2}b$$

Hence area of triangle =  $m \cdot h$

Post Assessment Element VI

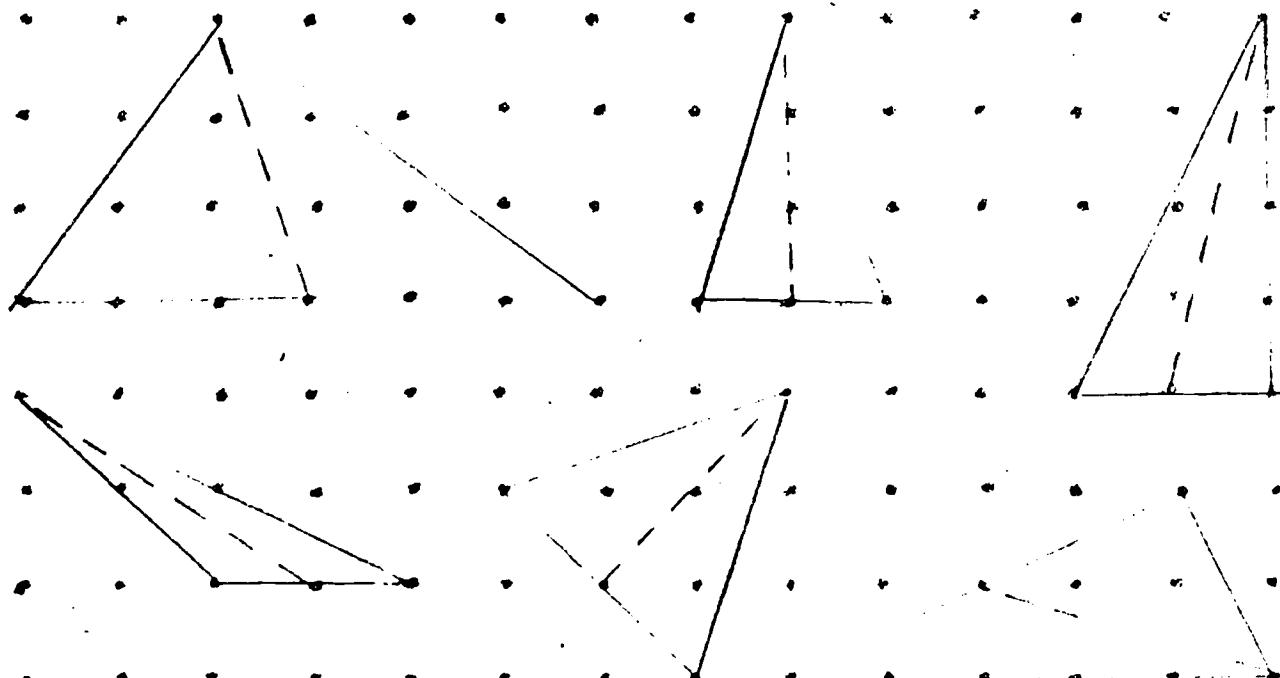
Task III

a. Some possible constructions:



Task III (cont'd.)

b. Some possible constructions:



The two triangles formed in each case are equal in area.

Hypothesis: If a median is drawn in a triangle, it divides the triangle into two triangles having equal area.

Justification: In each instance the two triangles formed have equivalent bases and the same height. Hence their areas are equal.

Post-Assessment for Element VII

Task I

3 rolls

Task II

Yes

Circumference of base of lampshade is 36"  
Surface area of lampshade = 648 square inches

Task III

Table 7.12

	Perimeter	Approximate Area
Triangle I	12	6.0
II	12	4.8
III	12	7.5
IV	12	5.8
V	12	6.4
VI	12	4.3

Triangle V has largest area.

Triangle V is equilateral and equiangular.

Quadrilateral I	12	5.0
II	12	8.0
III	12	9.0
IV	12	6.4
V	12	6.6
VI	12	8.4
VII	12	8.3
VIII	12	5.7

Quadrilateral III has largest area.

Quadrilateral III is equilateral and equiangular.

Pentagon I	12	9.3
II	12	10.2
III	12	6.9

Pentagon II has largest area.

Pentagon II is equilateral and equiangular

Task III (cont'd.)

Table 7.12 (continued)

	perIMeter	Approximate area
Hexagon 1 II	12 12	16.4 8.6
Circle	12	11.3

a. The perIMeters of all the figures are equal.

b. The areas of all the figures vary.

c. In each category the plane figure which is both equilateral and equiangular has the largest area.

d. The circle

e. Possible hypotheses might include:

- 1) Plane figures which have the same perIMeter may have different areas.
- 2) If polygons have the same number of sides and the same perIMeter, the polygon which has the largest area is both equilateral and equiangular.
- 3) As the number of sides of polygons which are both equilateral and equiangular increases, the area increases.
- 4) Of all plane figures which have the same perIMeter, the circle has the largest area.

f. Possible experimentation might include:

- (1) Creating other plane figures each with perIMeter of 12 and find their areas.
- (2) Creating sets of plane figures each with perIMeter of 18 (or 24,...) and find their areas.